Highly Rarefied Planar Jet Impingement On a Flat Plate

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Abstract. This paper reports a study on a highly rarefied gaseous jet plume flows out of a two-dimensional slit and impinges on a flat plate which is set vertically to the plume flow direction. It includes some analytical collisionless flow properties for a free plume and the impingement on the plate. This study yields the maximum value of the shear stress and the location for a two-dimensional jet impinging on a flat plate. Numerical simulation results obtained with the direct simulation Monte Carlo method validate the analytical collisionless flow solutions.

Keywords: Rarefied gas flows, plume impingement, shear stress
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1. INTRODUCTION

Rarefied jet flow impingement on a flat plate have many important applications, such as: 1) thin film deposition process inside a vacuum chamber, where high speed carrier gas carries metal powder and impinges on a flat plate [1]; 2) rocket plume impinges on a spacecraft or solar panels [2]; 3) the interactions of rocket plume and lunar ground during a lunar-landing mission [3]. Among those examples, we are interested in the flowfield and the properties on the plate surface. Especially, the pressure and shear stress distributions on the plate are helpful for some applications including: 1). computing the drag force on solar panel surfaces; 2). determining the lunar ground breaking up position and the beginning of sand lifting up.

This work presents some results on a highly rarefied two-dimensional jet plume flow impinging on a flat plate. The past related work can be classified into two categories, i.e. free collisionless plume flows, and rarefied plume jet impinging on a flat plate. For the first part, usually a rarefied plume jet is modeled by assuming free molecular flows with a nonzero, uniform average exit velocity U_0. As pointed out by Woronowicz [4] for high speed plume flows, even the number density at the exit can be high, the relative velocity is very small, as such, intermolecular collisions happen very rarely. Narasimha [5] obtained the exact solutions of density and velocity distributions for a free molecular effusion flow; Brook [6] reported the density field of free molecular flow from an annulus, to study the gas leakage from a spacecraft hatch. Lilly *et al* [7] reported their work on measurement and computation of rarefied mass flow and momentum flux through short tubes. For the case of free molecular flows with a nonzero average velocity, usually the problems are very complicated [8]. Two previous studies [9, 10] provided detailed macroscopic solutions of collisionless plume flows; and they serve as the foundation for the rarefied plume jet impingement problem discussed here. For the second part, there are some numerical and experimental results as well. For example, Bradshaw [11] reported measurements of velocity magnitude and direction, static pressure and skip friction for a case of a circular air jet impinging normally on a flat surface. Sathian [12] reported an measurement of shear stress determination due to impingement of low-density free jets on a flat plate. Maddox [13] reported a computation method to determine the drag and heat flux by under expanded plumes to adjacent surfaces. Kannenberg and Boyd [14] ingeniously proposed some formula for plume impingement on a plate surface with a hypersonic limit assumption.

For a lunar or Martian landing mission with a retro-rocket, the interactions among rarefied rocket plume, crater and dust is one of the most challenging tasks[15]. Due to the special environment, to perform experimental study with accurate parameters is almost impossible, and we have to rely on numerical simulations and analytical studies to aid our understanding on this problem.

2. TWO-DIMENSIONAL RAREFIED JET IMPINGEMENT ON A FLAT PLATE

This section discusses the problem of a rarefied two-dimensional collisionless jet impinging on a flat plate. Besides the macroscopic flowfield properties, we are specifically interested in the slip velocity, pressure and shear stress on the plate surface.

Rarefied plume jet, with known density ρ_0, average velocity U_0, and temperature T_0, is fired from a nozzle of a width D. One plate is placed at a distance of L to the nozzle, Figure 1 illustrates the problem.

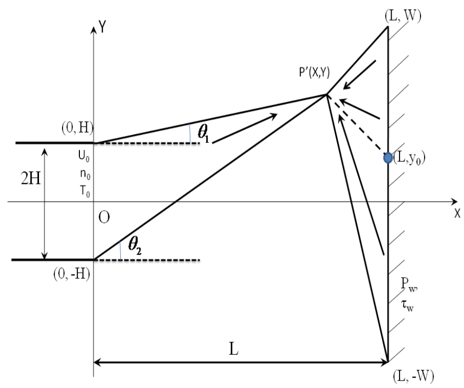

FIGURE 1. Illustration of the problem.

2.1. Two-dimensional Free Molecular Jet Plume Expanding into Vacuum

In our past study[9], we consider the velocity space for a point (X,Y) in front of a planar slit, the velocity distribution at the point follows a Maxwellian distribution. Collisionless gas flows out of the slit with a macroscopic mean velocity U_0. The number density and velocities at any point (X,Y) in front of the slit were derived [9]:

$$\begin{aligned} n_1(X,Y) = & \frac{\exp(-M_0^2)}{2\pi}(\theta_2-\theta_1)+\frac{1}{4}\left(\mathrm{erf}(M_0\sin\theta_2)-\mathrm{sign}(\theta_1)\mathrm{erf}(M_0\sin|\theta_1|)\right) \\ & +\frac{M_0}{2\sqrt{\pi}}\int_{\theta_1}^{\theta_2}\exp(-M_0^2\sin^2\theta)\cos\theta\,\mathrm{erf}(M_0\cos\theta)d\theta \end{aligned} \tag{1}$$

$$\begin{aligned} \frac{U_1(X,Y)}{\sqrt{2RT_0}} = & \frac{\exp(-M_0^2)}{2n_1\pi}\left[\int_{\theta_1}^{\theta_2}\left(\frac{\sqrt{\pi}}{2}\exp(M_0^2\cos^2\theta)\cos\theta[1+\mathrm{erf}(M_0\cos\theta)]\right)d\theta+\frac{M_0(\theta_2-\theta_1)}{2}\right. \\ & \left.+\frac{M_0(\sin(2\theta_2)-\sin(2\theta_1))}{4}+M_0^2\sqrt{\pi}\int_{\theta_1}^{\theta_2}\left(\cos^3\theta[1+\mathrm{erf}(M_0\cos\theta)]\exp(M_0^2\cos^2\theta)\right)d\theta\right] \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{V_1(X,Y)}{\sqrt{2RT_0}} = & \frac{1}{4\sqrt{\pi}n_1}\left[\exp(-M_0^2\sin^2\theta_1)\cos\theta_1[1+\mathrm{erf}(M_0\cos\theta_1)]\right. \\ & \left.-\exp(-M_0^2\sin^2\theta_2)\cos\theta_2[1+\mathrm{erf}(M_0\cos\theta_2)]\right] \end{aligned} \tag{3}$$

where the subscript “$_1$” represents the free plume expansion solutions.

More details and validation work about n_1, U_1, V_1 and p_1 solutions are available in the previous study [9], [16]. Because the flow is rarefied, it is appropriate to utilize the direct simulation Monte Carlo (DSMC) method [17] to validate the analytical results.

2.2. Collisionless Plume Impingement Problem: Plate Properties

For the plume impingement problem, the number density on the plate consists of two terms:

$$n_2(L,y) = n_1(L,y) + n'_w(y) \tag{4}$$

where the left hand side term $n_2(L,y)$ is the density solution for the plume impingement problem. At the right hand side, $n'_w(y)$ is the new density factor contributed from the plate, and $n_1(L,y)$ is the free plume solution [16].

It is evident that the velocity $U_2(L,y)$ is zero along the plate surface due to the non-penetration condition, and we utilize this condition to determine $n'_w(y)$. Suppose the velocity distribution function for those reflected particles is:

$$f_w(y) = n_w(y)(\beta_w/\pi)\exp[-\beta_w(u^2+v^2)] \tag{5}$$

Then from an integration with u as the moment, we can obtain:

$$n_1(L,y)U_1(L,y) = \frac{n_w\sqrt{T_w}}{\sqrt{2\pi}} \tag{6}$$

By integrating Eqn.(5) with the left half of the velocity space, we obtain the contributions from the plate to $n_2(L,y)$:

$$n'_w(y) = \sqrt{\frac{\pi}{2}}\frac{n_1(L,y)U_1(L,y)}{\sqrt{T_w}} \tag{7}$$

As for the slip velocity at the plate, it is obtained with the following relation,

$$V_2(L,y) = \frac{V_1\sqrt{T_w}}{\sqrt{T_w}+\sqrt{\pi/2}U_1} \tag{8}$$

The contribution to the slip velocity from the plate velocity distribution, Eqn.(5), is zero because we assume the plate is completely diffuse. This slip-velocity is important for simulations of dust particle-gas flow with the Discrete Element Method (DEM) because it provides the crucial input data.

The temperature at locations very close to the plate are:

$$\begin{aligned} T_2(L-,y) &= -\frac{V_2^2}{3R} + \frac{n_w T_w}{2n_2(L,y)} + \frac{T_0\exp(-M_0^2)}{6n_2\pi}\Big[3\Delta\theta + M_0^2(\Delta\theta + \tfrac{1}{2}[\sin(2\theta_2)-\sin(2\theta_1)]) \\ &+2\sqrt{\pi}M_0\int_{\theta_1}^{\theta_2}(2\cos\theta + M_0^2\cos^3\theta)\exp(M_0^2\cos^2\theta)(1+\mathrm{erf}(M_0\cos\theta))d\theta\Big] \end{aligned} \tag{9}$$

No matter how close to the plate a point locates, the temperature and pressure definitions are averaged properties along the x-, y- and z- directions. The temperature is important to evaluate the viscosity at locations very close to the plate, and it is an input parameter for a DEM simulation as well.

Since the plate normal direction is defined on the plate surface, we can obtain the shear stress on the plate with a simpler format:

$$\begin{aligned} \tau_{xy} &= \frac{mn_0RT_0}{2\pi}\Big[(\cos^2\theta_1 - \cos^2\theta_2)\exp(-M_0^2) + M_0\cos^3\theta_1\exp(-M_0^2\sin^2\theta_1)\sqrt{\pi}[1+\mathrm{erf}(M_0\cos\theta_1)] \\ &-M_0\cos^3\theta_2\exp(-M_0^2\sin^2\theta_2)\sqrt{\pi}[1+\mathrm{erf}(M_0\cos\theta_2)]\Big] \end{aligned} \tag{10}$$

The location with the maximum shear stress is of special interest, for example, the lunar ground breaks off firstly there. We can obtain such a location from Eqn.(10) for a two-dimensional case. With some assumptions, we obtain the following simple formula for the critical location with the maximum shear stress:

$$\frac{y_{crit}}{D} = \sqrt{\frac{(L/D)^2}{3M_0^2} - \frac{1}{12}} \tag{11}$$

this result is based on one assumption that L/D is large. In one NASA's experiment, it is found the largest ground shear stress happens at a location proportional to L/D, while inversely proportionably to the nozzle exit Mach number [18]. Equation (11) provides a strong support for this result. With the above simple result the simplified maximum shear stress value is:

$$\frac{\tau_{max}}{\rho_0 U_0^2/2} = \frac{1}{2\sqrt{3\pi}}[1+\mathrm{erf}(M_0)][1+\frac{3}{2M_0^2}]\frac{D}{L} \tag{12}$$

3. VALIDATIONS

We simulate a collisionless jet impinging on a plate with the nozzle exit Mach number $M_0 = 2$. The gas is Argon, the nozzle diameter is $D = 0.2$ m and the nozzle height $L = 2.0$ m.

Figure 2 shows several pressure profiles on the plate surface, since the plate normal direction is determined, the pressure for this situation only includes the components related to the normal direction. The analytical treatment and DSMC simulation have consistent results.

Figure 3 shows the plate shear stress result. The analytical and the DSMC simulation results are virtually identical, the locations with the maximum shear stress are very consistent. The simplified explicit location formula, Eqn.(11) anticipates a location $y_{critical}/D = 2.88$, while a detailed examination of the DSMC simulation results for shear stress curve shows the location is $y_{critical}/D = 2.82$. The simplified maximum shear stress formula, Eqn.(12) predicts a value of 0.0446 while a detailed examination on the DSMC simulation results shows the corresponding value is 0.05088. As such, we conclude that the concise equations, Eqns.(11) and (12), are accurate. Figure 4 shows the corresponding temperature distributions along near-plate locations, i.e., still in the flowfield. The simulation results have good agreement as well.

Figure 5 shows the normalized maximum shear stress on the plate surface for different Knudsen numbers, from this figure we conclude that as the Kn number increases, the values of maximum shear stress from the DSMC become closer to the simplified formula, Eqn.(12), while the DSMC and analytical results are in good agreement for different Kn. Figure 6 shows the normalized critical location of the maximum shear stress on the plate for different Knudsen numbers with $L = 2$ m. We normalized the location of the maximum shear stress with the diameter of the nozzle, $D = 0.2$ m. It is evident that as the Kn number increases, the location with the maximum shear stress on the plate becomes closer to the simplified formula Eqn.(11).

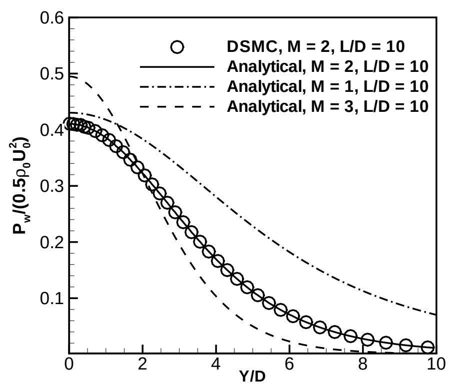

FIGURE 2. Profiles of normalized pressure on the plate surface, $P(L-,y)/(\rho_0 U_0^2/2)$, collisionless, $L/D = 10$.

4. SUMMARY

We have analyzed the problem of rarefied two-dimensional jet flows impinging on a flat plate, and validated the results. First we revisited the complete solutions for collisionless free plume flow expanding into vacuum. By adding a plate, we obtained some properties very close to the plate and the on-plate shear stress, pressure distributions and slip velocities. For collisionless flows, the analytical results are almost identical to the DSMC simulations results.

For the large range of rarefied regimes with $Kn \geq 1$, we can adopt the analytical results presented in this paper for fast engineering estimations with very minor discrepancies. Even though the analytical results are complex, the detailed exact format permit us to study different factors systematically. One example is the critical plate location with the largest shear stress, which is critical for rocket plume and lunar ground interactions, and our study provides strong supports to NASA's experimental report [18].

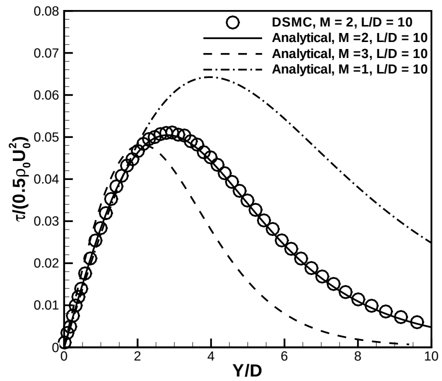

FIGURE 3. Profiles of normalized shear stress on the plate surface, $\tau_{xy}/(\rho_0 U_0^2/2)$, collisionless, $L/D = 10$.

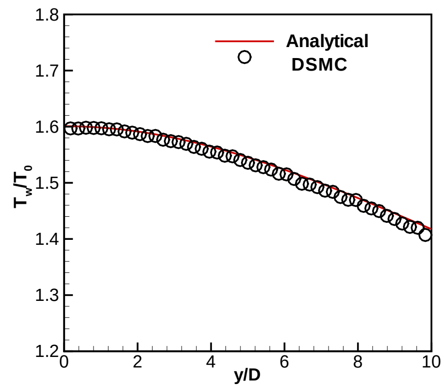

FIGURE 4. Profiles of normalized near-plate temperature, T_w/T_0, collisionless, $M_0 = 2$, $L/D = 10$.

ACKNOWLEDGMENTS

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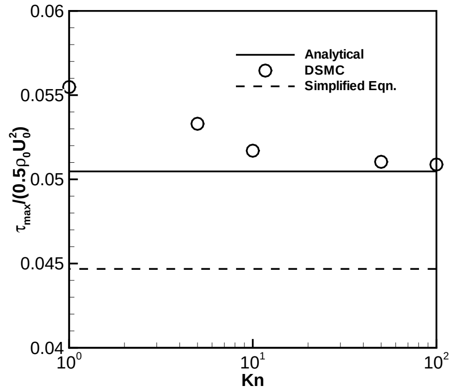

FIGURE 5. Normalized maximum shear stress on the plate surface for different Kn number, $M_0 = 2$, $L/D = 10$.

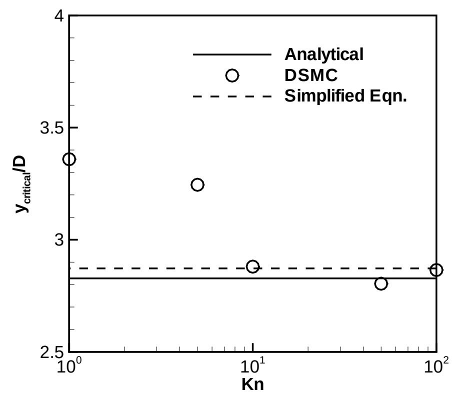

FIGURE 6. Normalized critical plate locations with the maximum shear stress for different Kn number, $M_0 = 2$.

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